4.1 Antiderivatives

WCID? I can take the antiderivative of a function

A. Take the derivative of each of the following:

1. $f(x) = x^3$ $f'(x) = 3x^2$

2. $f(x) = x^3 + 7$ $f'(x) = 3x^2$

3. $f(x) = x^3 - 125$ $f'(x) = 3x^2$

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What do you notice?

B. We will start to do derivatives backwards – called antiderivatives

1. Many functions have the same derivative. Usually they differ by a constant

2. When we take an antiderivative, it will have this constant shown by "C"

3. Notation: $\int f(x) = F(x) + C$

$\int f(x)dx = F(x)+C$

4. Since "C" can be anything, this is called a general solution

C. How to take an antiderivative:
→How do you take a derivative????

**JUST DO THIS BACKWARDS!!!!

1. Increase the power

2. Divide by the new power

Ex: $f(x) = x^3$ $\int x^3 dx = \frac{x^4}{4} + C$

Ex: $f(x) = x^4 + 2x^2 + 7x$

$\int x^4 + 2x^2 + 7x \, dx$

$\frac{1}{5}x^5 + \frac{2x^3}{3} + \frac{7x^2}{2} + C$

Ex: $f(x) = 3x^5 - 4x^3 - 11x^2$

$\int 3x^5 - 4x^3 - 11x^2 dx = \frac{3x^6}{6} - \frac{4x^4}{4} - \frac{11x^3}{3} + C$

$\frac{x^6}{2} - x^4 - \frac{11}{3}x^3 + C$

Ex: $f(x) = \cos x$ $\rightarrow$ *THINK:* What did you take the derivative of?????

$\int \cos x \, dx = \sin x + C$

Ex: $f(x) = \sec^2 x$

$\int \sec^2 x \, dx = \tan x + C$

D. Special Cases

1. $\int 0\,dx = C$

2. $\int k\,dx = kx + C$ $\int 5\,dx = 5x + C$

3. $\int (f(x) \pm g(x))\,dx = \int f(x)\,dx \pm \int g(x)\,dx + C$

4. $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \rightarrow$ THIS IS WHAT WE DID EARLIER

5. $\int kf(x)\,dx = k\int f(x) \rightarrow$ Constant Multiple Rule

Examples: Take the Antiderivative of each of the following

$\int x^{-4}\,dx$ $\quad \frac{x^{-3}}{-3} + C \quad -\frac{1}{3x^3} + C$

$\int \left(x^3 + \cos x\right)dx$ $\quad \frac{x^4}{4} + \sin x + C$

$\int \left(4\sec x \tan x + 3x^2\right)dx$

$4\sec x + x^3 + C$

$$\int (x+3)(x-5)\,dx$$

$$\int (x^2 - 2x - 15)\,dx = \frac{x^3}{3} - x^2 - 15x + C$$

$$\int x^2(1-x^3)\,dx$$

$$\int (x^2 - x^5)\,dx = \frac{x^3}{3} - \frac{x^6}{6} + C$$

$$\int \left(\frac{x+1}{\sqrt{x}}\right)dx \qquad \frac{2x^{3/2}}{3} + \frac{2x^{1/2}}{1} + C$$

$$\int \left(\frac{x^1}{x^{1/2}} + \frac{1}{x^{1/2}}\right)dx$$

$$\int \left(x^{1/2} + x^{-1/2}\right)dx = \frac{x^{3/2}}{3/2} + \frac{x^{1/2}}{1/2} + C$$

HW: p.255 1,2, 15-33 odd, 35-40