4.5A Substitution

A. $\int (2x+3)\,dx = x^2+3x+C$

$\int (x^2+4x+\cos x)\,dx$

$\frac{x^3}{3}+2x^2+\sin x+C$

1. $\int (x^2+3x+4)^8 (2x+3)\,dx$

$u = x^2+3x+4$

$du = (2x+3)\,dx$

$\int u^8\,du$

$\frac{u^9}{9} + C$

$\frac{(x^2+3x+4)^9}{9} + C$

Steps

1. Define u and du
2. Manipulate so all x's are gone
3. Integrate
4. Return to x

Ex.

$$\int 2x\sqrt{1+x^2}\,dx$$

$$\int 2x(1+x^2)^{1/2}\,dx$$

$u = 1+x^2$

$du = 2x\,dx$

$$\int u^{1/2}\,du$$

$$\frac{2u^{3/2}}{3} + C$$

$$\frac{2(1+x^2)^{3/2}}{3} + C$$

Ex. $\int \underline{5} \cos(5x)\, \underline{dx}$

$u = 5x$

$du = 5dx$

$\int \cos u \, du$

$\sin u + C$

$\sin(5x) + C$

Ex. $\int x(x^2+1)^{20}\,dx$

$u = x^2+1$

$\frac{du}{2} = \frac{2x\,dx}{2}$

$\frac{du}{2} = x\,dx$

$\int u^{20}\frac{du}{2}$

$\frac{1}{2}\int u^{20}\,du$

$\frac{1}{2}\frac{u^{21}}{21}+C$

$\frac{(x^2+1)^{21}}{42}+C$

$$\int x(2x^2-5)^{30}\,dx$$

$u = 2x^2 - 5$

$du = 4x\,dx$

$\frac{1}{4}\int u^{30}\,du$

$\frac{1}{4}\frac{u^{31}}{31} + C$

$\frac{(2x^2-5)^{31}}{124} + C$